

## A TWO-DIMENSIONAL ANALYSIS OF LAMINAR FLUID FLOW IN ROD BUNDLES OF ARBITRARY ARRANGEMENT\*

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**Abstract**—This paper presents an outline of an analytical method for calculating fully developed, longitudinal laminar flow in multiple connected regions as for instance in finite tube bundles. This method overcomes the limitation of previous investigations hitherto available and extends the analysis to problems of more than one rod ring. Results are given in form of velocity fields and the product  $f_i \cdot Re$  in dependence on pitch-to-diameter ratio, tube wall spacing, radial displacement of a rod ring and the angle of the characteristic symmetry segment.

### NOMENCLATURE

$a$ , nondimensional radius of rods;  
 $A_n, B_n$ , unknown coefficients in the general solution  
 $C_n^{(l)}$ , of the Poisson's equation;  
 $b$ , nondimensional radius of rings;  
 $D$ , nondimensional rod diameters (Fig. 4);  
 $D_h$ , hydraulic diameter,  $D_h = 4 \cdot \text{flow area/wetted perimeter}$ ;  
 $f_t$ , total friction factor;  
 $g_0^{(l)}$ , function, defined by equation (10);  
 $g_n^{(l)}$ , function, defined by equation (11);  
 $h_0^{(l)}$ , function, defined by equation (12);  
 $h_n^{(l)}$ , function, defined by equation (13);  
 $L^*$ , number of rings;  
 $m$ , number of peripheral rods, which are located on a ring;  
 $Q_F$ , nondimensional pressure gradients,  

$$Q_F = \frac{dP}{dz'}, \frac{R^2}{\mu \bar{v}};$$
 $P$ , nondimensional rod bundle pitch (Fig. 4);  
 $r$ , nondimensional radial coordinate referred to the center of the central rod;  
 $R$ , radius of the tube;  
 $Re$ , Reynolds number,  $Re = \frac{\rho_F \bar{v} D_h}{\mu}$ ;

$u$ , nondimensional axial velocity,  $u = v/\bar{v}$ ;  
 $U_{0,n}$ , periodic harmonic functions (real part of functions  $W_{0,n}^{(l)}$ );  
 $v$ , axial velocity;  
 $\bar{v}$ , average axial velocity;  
 $W_{0,n}^{(l)}$ , analytic functions defined by equations (8) and (9);  
 $W$ , nondimensional wall spacing (Fig. 5);  
 $z$ , complex variable,  $z = r \cdot e^{i\theta}$ ;  
 $z'$ , axial coordinate.

### Greek symbols

$\alpha$ , angle of the rotation of the symmetry axes with respect to the x-axis (Fig. 2);  
 $\theta$ , angular coordinates referred to the center of peripheral rods;  
 $\mu$ , dynamic viscosity of the fluid;  
 $\rho$ , nondimensional radial coordinates referred to the center of the peripheral rods;  
 $\rho_F$ , fluid density;  
 $\bar{\tau}$ , average wall shear stress;  
 $\varphi$ , angular coordinate referred to the center of the central rod;  
 $\varphi^*$ , angle of the symmetry segments,  $\varphi^* = \pi/m$ .

### Subscripts

$K$ , rod ring;  
 $l$ , rod ring; peripheral rods;  
 $l, n, p, s$ , summation indices, integers;  
 $p$ , particular;  
 $r$ , regular;  
 $s$ , singular;  
 $z$ , central rod.

\*Dedicated to Prof. h. Prof. Dr. sc. techn. Romano Gregorig for his 65th birthday.

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## 1. INTRODUCTION

THE MOST frequent type of fuel arrangement used in nuclear power reactors is a bundle of fuel rods located inside a round or hexagonal tube in which the cooling medium flows in-line. In most of these bundles the rods are in a regular array of square or triangular pitch. It was only with the start-up of the application of nuclear energy that a detailed knowledge of the hydrodynamics and energy transfer in these geometries became more and more important, although tube bundle heat exchangers have long been in use.

Therefore, it is not surprising to note that most of the previous and today's work is done with special emphasis to the nuclear field. In the first beginning, the laminar flow field in an infinite bundle which consists of a regularly repeated array was given by Sparrow *et al.* [1] and was reexamined later on by Subbotin *et al.* [2] with different mathematical tools. Axford [3] and Dwyer and Berry [4] resolved the problem in conjunction with the solution of the heat-transfer problems. Leonard and Lemlich [5] presented the solution for the extreme case of touching rods. Due to the inherent symmetry of the rod arrangement only a small characteristic flow field region needed to be analyzed. The domain under consideration is bounded partially by lines which do not coincide with surfaces of an orthogonal coordinate system. Hence, the boundary conditions on these lines has to be satisfied pointwise by means of a boundary collocation technique. However, this procedure is accompanied by handling large systems of algebraic equations and in view of the limited computer storage at this time the analyses were restricted to small flow regions. Therefore, the only attempt to study the additional wall effect was given by Schmid [6] who analyzed the variation of the volumetric flow rate in a semi-infinite square bundle, which was bounded by one plane wall.

At the same period, analytical methods as for instance the periodically harmonic Howland functions [7] came to light again in conjunction with the solution of problems in finite tube bundles in which the rods are concentrically arranged on a ring around a central rod. This geometry allows to study extensively the influence of the tube wall on the flow field for the first time. Axford [8] used the algorithm given by Howland and proposed a method of solution but did not give any results. The advantage of this method lies in the fact that the boundary condition can be continuously satisfied. Therefore, Chen [9] used the same method later on in his extension to eccentrically shifted rod arrangement within the tube wall. In addition, he solved the heat-transfer problem, too. On the other hand, Min [10, 11] made use of a mapping function, transformed the complex potential and satisfied pointwise

the boundary conditions on the peripheral rods. He reported a great amount of results.

Although the above mentioned analyses give some inside into the hydrodynamics and energy transfer in finite tube bundles they are of limited practical application because they are all restricted to bundles with only one ring of rods around a central rod. Additionally, these studies did not meet the requirements of the current fuel element bundle design in the nuclear industry, due to the curved tube wall and the circular arrangement of the rods. In view of the fact, that most of the variations in momentum and energy transport occur nearby the tube wall and in the domain of intersecting plane tube walls, i.e. in the corner region, the main interest nowadays concentrates on analyzing the side and corner subcells. The first step in this direction was made by Gunn and Darling [12] who used an overall numerical method to calculate velocity fields and pressure drops in the characteristic subchannels of a finite 4-rod bundle in a square array. Rehme [13, 14] took the same approach for the solution of a 7-rod bundle in a triangular array. However, he quoted [14] that a finite difference solution for bundles with more than seven rods results in an enormous consumption in computer storage and time. Therefore, he proposed a subchannel analysis based on the superposition of the local subcell solutions. He compared his values for the product of  $f_i \cdot Re$  with values given by Gunn and Darling [12] and with the only experimental data, hitherto available, presented by Galloway and Epstein [15] for 16-rod bundles in a square array and 19-rod bundles in a triangular array for different wall spacings. Rehme's data resulting from local calculations are in good agreement with those reported in [12]. However, the comparison with the 16- and 19-rod bundle data is not correct, because he compared the wrong data with respect to the wall spacing. A careful inspection of the data shows at least a difference of 10 per cent which is in contrast to Rehme's conclusion. This shows clearly the uncertainty in using subchannel analyses.

To overcome the difficulties in analyzing for more than 7-rod bundles and to extend the limitations of studying only one ring problems, Mottaghian and Wolf [16] proposed an analytical method which enables one to prove the validity of the subchannel codes based on a lumped parameter description of the transport processes in the rod bundles.

## 2. FORMULATION OF THE PROBLEM

Whereas all previous investigations are limited to one ring problems, this analysis considers a tube bundle consisting of an arbitrary number of rods with different radii  $a_l$  ( $l = 1, 2, \dots, L^*$ ) placed on concentric rings of radii  $b_l$  around a central rod with the radius  $a_2$ . The

only limitation of this analysis is the assumption that the bundle has a characteristic symmetry with respect to the angular direction. This segment is shown in Fig. 1 together with all geometrical, nondimensional variables. The  $(r, \varphi)$ -polar-coordinate systems refers to the center of the central rod, whereas the  $(b_l \rho_l, \theta_l)$ -polar-coordinate systems are associated with the centers of the individual peripheral rods.

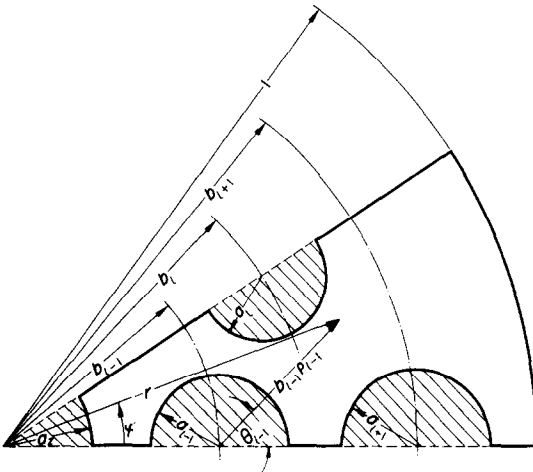


FIG. 1. Symmetry segment of a finite rod bundle.

The flow field is described generally by the equation of continuity and the equation of motion which under the assumptions of (1) steady state; (2) fully-developed, isothermal incompressible Newtonian laminar flow and (3) pressure as the only force acting on the fluid reduce to

$$\nabla^2 u = Q_F. \tag{1}$$

The boundary conditions which has to be satisfied are: zero velocity at every surface in the bundle

$$u = 0; \tag{2}$$

the derivative of the velocity field with respect to the azimuthal coordinate must be zero along every axis of symmetry

$$\frac{\partial u}{\partial \varphi} = 0. \tag{3}$$

### 3. METHOD OF SOLUTION

Although the equation (1), which is of the Poisson type, is simple in nature, the solution of the problem is complex in view of satisfying simultaneously all boundary conditions, equation (2), at the surfaces. The only way to do this economically seems to be the method of superposition of solutions. Basically the overall solution is split into three major parts

$$u = Q_F(u_p + u_r + u_s). \tag{4}$$

The first part is the wellknown particular solution of the Poisson equation

$$u_p = \frac{1}{4} r^2, \tag{5}$$

whereas the second part constitutes the regular solution and follows to be with respect to the symmetry condition, equation (3),

$$u_r = A_0 + B_0 \ln r + \sum_{n=1}^{\infty} A_n r^{nm} \cos(nm\varphi) + \sum_{n=1}^{\infty} B_n r^{-nm} \cos(nm\varphi). \tag{6}$$

This solution represents the flow field in the annulus alone if the central rod is present. If there is no central rod then with  $B_0 = B_n = 0$ , equation (6) constitutes the solution for a tube alone.

The third part in equation (4) is the singular solution and represents the influence of the rods in the rings on the whole flow field. For  $u_s$  the following double serie is set up

$$u_s = \sum_{l=1}^{L^*} \sum_{n=1}^{\infty} C_n^{(l)} U_n^{(l)}, \tag{7}$$

in which the first summation represents the number of rod rings. The extension to a double sum as shown in equation (7) is the basic principle underlying the following analysis. The periodic and harmonic functions  $U_n^{(l)}$  represent the real part of the analytic functions

$$W_0^{(l)}(z) = -\ln \prod_{p=0}^{m-1} (z - z_{pl}) \tag{8}$$

$$= -\ln(z^m - z_0^m) \tag{8a}$$

and

$$W_n^{(l)}(z) = \frac{b_l^n}{(n-1)!} \frac{d^n W_0^{(l)}(z)}{db_l^n} \quad l = 1, 2, \dots, L^* \quad n = 1, 2, \dots \tag{9}$$

The midpoints of the rods  $z_{pl}$  can be formulated as follows

$$z_{pl} = b_l \exp \left[ i \left( 2p \frac{\pi}{m} + \alpha_l \right) \right] \quad (\text{see Fig. 2}).$$

The functions  $W_0^{(l)}(z)$  have logarithmic singularities in the origins of the rods and the functions  $W_n^{(l)}(z)$  have poles of order  $n$  at  $z = z_{pl}$ .

In view of satisfying the non-slip condition on the surface of the central rod and additionally on the channel wall, the overall solution has to be represented in the  $(r, \varphi)$ -coordinate system. Both, the particular and regular solutions are in the right form as can be seen by inspection of the equations (5) and (6), whereas the singular solution has to be transformed. To do this the analytic functions  $W_0^{(l)}$  and  $W_n^{(l)}$  must be developed into infinite series in terms of  $z$ . In view of the above mentioned singularities the development into series

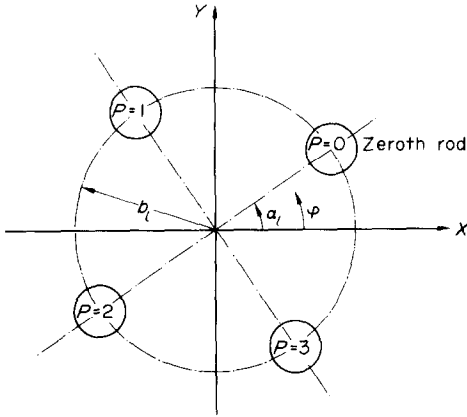


FIG. 2. Rotation of the symmetry axis with respect to the x-axis.

must be done for two domains, i.e.  $|z| < |z_{pl}|$  and  $|z| > |z_{pl}|$ .

After the separation of the real and imaginary parts of the functions  $W_0^{(l)}$  and  $W_n^{(l)}$  the following results hold for the functions  $U_n^{(l)}$  in the different regions

$r < b_l$ ,

$$U_0^{(l)} = -m \ln(b_l) + \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{r}{b_l}\right)^{ms} \cos[ms(\varphi - \alpha_l)] = g_0^{(l)}(r, \varphi) \tag{10}$$

$$U_n^{(l)} = (-1)^n m \left\{ 1 + \sum_{s=1}^{\infty} \binom{ms+n-1}{n-1} \left(\frac{r}{b_l}\right)^{ms} \cos[ms(\varphi - \alpha_l)] \right\} = g_n^{(l)}(r, \varphi) \quad n = 1, 2, \dots \tag{11}$$

$r > b_l$ ,

$$U_0^{(l)} = -m \ln(r) + \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{b_l}{r}\right)^{ms} \cos[ms(\varphi - \alpha_l)] = h_0^{(l)}(r, \varphi) \tag{12}$$

$$U_n^{(l)} = m \sum_{s=1}^{\infty} \binom{ms-1}{n-1} \left(\frac{b_l}{r}\right)^{ms} \cos[ms(\varphi - \alpha_l)] = h_n^{(l)}(r, \varphi) \quad n = 1, 2, \dots \tag{13}$$

Introducing these equations into equation (7) results in the following forms of the singular solution, depending upon the different regions in the tube bundle:

For the region in between the first ring of rods, i.e.

$$0 \leq r < b_1 \tag{14}$$

$$U_s = \sum_{l=1}^{L^*} \left( C_0^{(l)} g_0^{(l)} + \sum_{n=1}^{\infty} C_n^{(l)} g_n^{(l)} \right).$$

For the whole region between the first and the  $L^*$ -th

ring of rods, i.e.

$$b_K < r < b_{K+1},$$

$$U_s = \sum_{l=1}^K \left( C_0^{(l)} h_0^{(l)} + \sum_{n=1}^{\infty} C_n^{(l)} h_n^{(l)} \right) + \sum_{l=K+1}^{L^*} \left( C_0^{(l)} g_0^{(l)} + \sum_{n=1}^{\infty} C_n^{(l)} g_n^{(l)} \right) \tag{15}$$

$K = 1, 2, \dots, L^* - 1.$

For the region in between the last ring of rods and the tube wall

$$b_{L^*} < r \leq 1,$$

$$U_s = \sum_{l=1}^{L^*} \left( C_0^{(l)} h_0^{(l)} + \sum_{n=1}^{\infty} C_n^{(l)} h_n^{(l)} \right). \tag{16}$$

After these transformations of the singular solution, the general solution now satisfies the differential equation (1) as well as the boundary conditions, equations (2) and (3), at the central rod surface and the tube wall, continuously. With the solution in this form it is generally possible to satisfy the boundary condition at any arbitrary surface in the domain between the central rod and the tube wall with the help of the point-matching technique or the boundary least square-method. However, in view of the resulting errors of the calculations with these methods on the boundary surfaces it is more advantageous in case of circular regions to perform an additional transformation to satisfy simultaneously the boundary conditions on the peripheral rods, too. For this reason, the general solution has to be transformed term by term from the  $(r, \varphi)$ -polar coordinate system into the  $(b_l \rho_l, \theta_l)$  polar coordinate systems of the individual peripheral rods. After these manipulations the solution is complete. Setting this solution into the boundary condition (2) results in an infinite system of coupled, inhomogeneous linear equations for the unknown coefficients in the regular as well as singular solutions. To solve this system it is advantageous to use on the one hand the orthogonality relations of trigonometric functions and on the other hand to reduce the system to one type of coefficient.

#### 4. RESULTS AND DISCUSSION

The effect of the rod arrangement, number of rods, rod spacings, tube wall spacings, radial rod displacements and rod diameters on the velocity field, local wall shear stress distributions, pressure gradients and total friction factors has been investigated.

In order to check the validity of the analysis and the computer program a special 7-rod bundle problem with two rings of three rods has been constituted. In the limiting case for  $b_2$  approaching  $b_1$ , this problem becomes consistent with the one ring 7-tube problem of previous investigations and the results show a very close agreement with those reported in [9, 11].

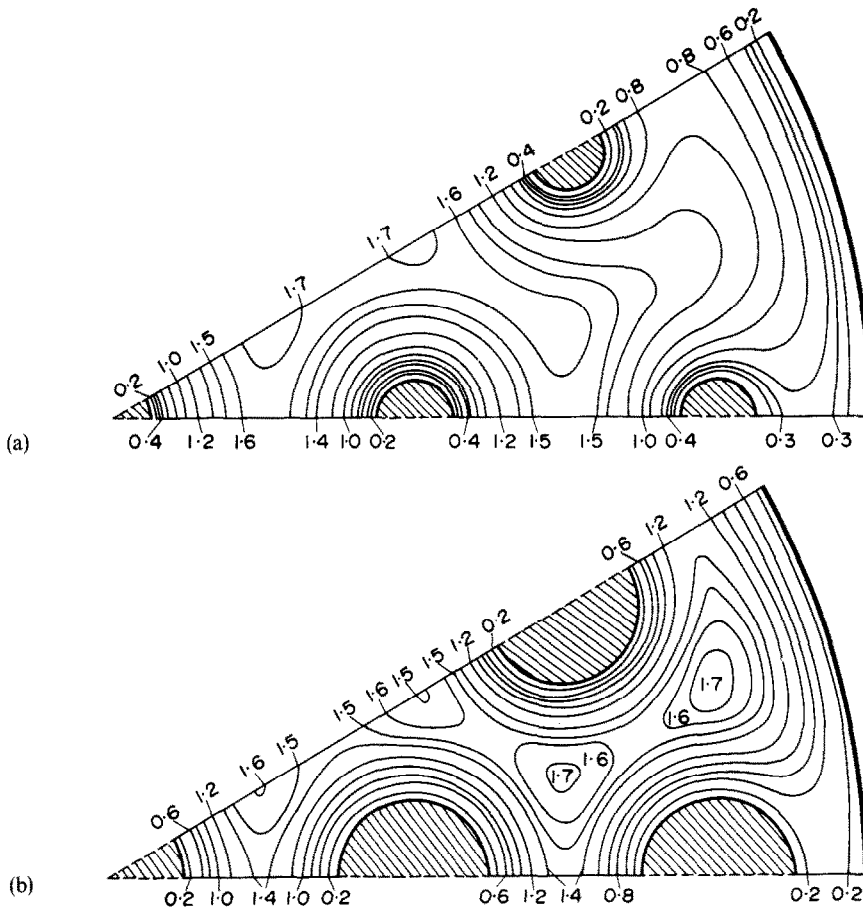


FIG. 3. Structure of isovels for laminar flow in finite rod bundles with 19 rods in triangular rod array.  $b_1 = 0.4; b_2 = 0.693; b_3 = 0.8$ ; (a)  $a_2 = a_1 = a_2 = a_3 = 0.05$ ; (b)  $a_2 = a_1 = a_2 = a_3 = 0.10$ .

For computational purposes, the aforementioned infinite system has to be truncated after  $N$  terms. The number of equations, i.e. number of coefficients, necessary to produce satisfactory convergence, primarily depends on the distances between two neighbouring rod rings. For the cases shown in the following figures the non-slip condition at the central rod and tube wall is satisfied within the order of  $10^{-15}$  whereas for the peripheral rods the value increases to  $10^{-6}$  which nevertheless seems to be a reasonable value, too.

Figure 3 shows the contours of constant velocity lines in 19-rod bundles with triangular rod arrangements for two different rod diameters. By comparison of the upper and lower part of Fig. 3 it is obvious that with increasing rod diameter the effect of the peripheral rods becomes greater on the flow around the central rod as well as on the flow around and between adjacent rods. The same holds for the flow pattern in the region between the outer rod ring and

the tube wall. Increasing the rod diameter results in a reduction of the flow area, and hence, for a fixed flow rate the local velocities in Fig. 3(b) are higher than those in Fig. 3(a).

The total friction factor  $f_t$  is defined as

$$f_t = \frac{\bar{\tau}}{\rho_F \frac{v^2}{2}}$$

whereas the average shear stress is given by

$$\bar{\tau} = -\frac{dp}{dz} \frac{D_h}{4}$$

In the following figures the product

$$f_t \cdot Re = -\frac{1}{2} Q_F D_h^2$$

is used to describe the pressure drop behavior in dependence of the aforementioned parameters.

In Fig. 4  $f_t \cdot Re$  is plotted vs the  $P/D$  ratio for 19-rod bundles arranged in a triangular array as shown by

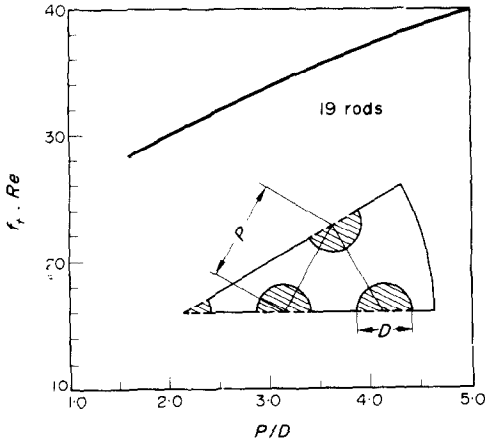


FIG. 4. Total friction factor times Reynolds number ( $f_t \cdot Re$ ) in dependence on pitch-to-diameter ratio ( $P/D$ ) for 19-rod bundles in triangular array.

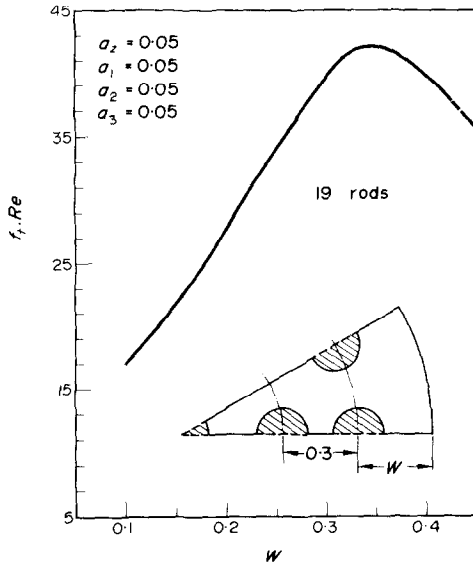


FIG. 5. Total friction factor times Reynolds number ( $f_t \cdot Re$ ) as a function of the wall spacing ( $W$ ).

the sketch. As a result  $f_t \cdot Re$  continuously increases with increasing  $P/D$  ratio.

As indicated in Fig. 5 the product  $f_t \cdot Re$  increases with increasing wall spacing, reaches a maximum nearby  $W = 0.35$  and then decreases continuously. The location of the maximum depends on the rod arrangement as well as on the rod diameter. The main result of this figure is that there exists a special location of the two rod rings in the region between the central rod and the tube wall for which the perturbation in the flow pattern is greatest in view of an overall pressure gradient.

The effect of varying the radius of the second rod ring  $b_2$  for the fixed positions of the first and third rod rings on  $f_t \cdot Re$  is shown in Fig. 6. For a fixed

flow rate  $f_t \cdot Re$  increases with increasing ring radius  $b_2$ , reaches a maximum value at  $b_2 = 0.63$  and then decreases monotonically with further increase of  $b_2$ . Therefore, in order to minimize  $f_t \cdot Re$  it seems to be more advantageous to put the additional rod into the first or third rod ring than to constitute a second rod ring.

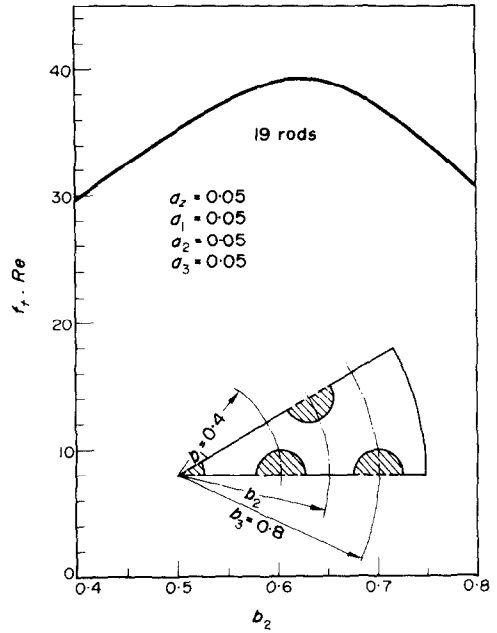


FIG. 6. Total friction factor times Reynolds number ( $f_t \cdot Re$ ) as a function of the radial displacements of the second rod ring with the radius ( $b_2$ ).

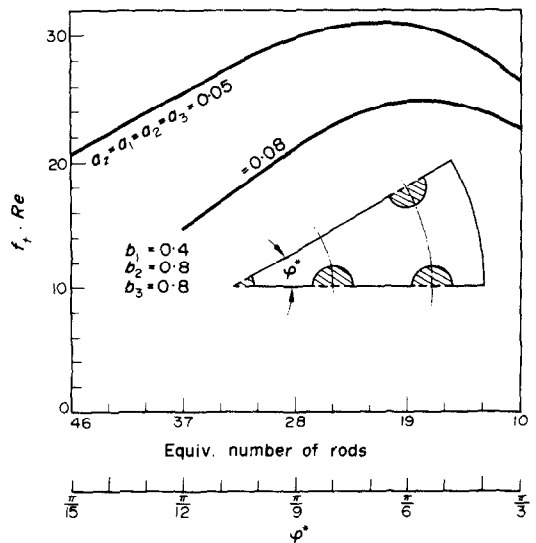


FIG. 7. Total friction factor times Reynolds number ( $f_t \cdot Re$ ) in dependence on the angle of the symmetry segment ( $\varphi^*$ ).

In Fig. 7  $f_t \cdot Re$  is plotted vs the angle of the symmetry segment. Decreasing this angle results in an increase of the total number of rods in the bundle. The rod diameter is chosen as an additional parameter. The curves show the existence of a maximum which decreases with increasing rod diameter and shifts slightly to greater angles. It is interesting to note that in the range of geometrical parameters considered here, the maxima occur in bundles with relative few rods.

### 5. CONCLUSION

The developed analytical method of solution constitutes the basis on which solution of the thermal problem is now possible for finite tube bundles with more than one rod ring. This additional effort seems to be necessary in view of a thermal-fluid dynamic optimization of such bundles. Additionally, such results may serve as input data and/or for proving the results of lumped parameter codes in view of the transport coefficients for the limiting case of laminar flow. As to the authors knowledge, the method of solution has some special features in reducing computer storage and computer time due to its analytical character.

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### UNE ANALYSE BIDIMENSIONNELLE DE L'ÉCOULEMENT LAMINAIRE D'UN FLUIDE DANS UNE GRAPPE DE BARRES AVEC UN ARRANGEMENT ARBITRAIRE

**Résumé**—On présente l'essentiel d'une méthode analytique de calcul d'un écoulement laminaire, longitudinal, pleinement développé dans des régions connectées de façon multiple comme, par exemple, dans des grappes de tubes. Cette méthode surmonte les limitations des études antérieures et étend l'analyse aux problèmes de plus d'un espace annulaire. Les résultats sont donnés sous la forme du champ de vitesse et du produit  $f_t \cdot Re$  en fonction du rapport pas/diamètre, de l'espacement des parois des tubes, du déplacement radial d'une barre et de l'angle du segment caractéristique de symétrie.

### EINE ZWEIDIMENSIONALE ANALYSE DER LAMINARSTRÖMUNG IN RUNDSTABBÜNDELN BELIEBIGER ANORDNUNG

**Zusammenfassung**—Es wird eine Methode vorgestellt, die es erstmals ermöglicht, ausgebildete Laminarströmung in endlichen Rundstabbündeln mit mehr als einem Ring von Stäben analytisch zu berechnen. Es werden zweidimensionale Geschwindigkeitsfelder und das Produkt  $f_t \cdot Re$  als Funktion des Stababstand-Stabdurchmesser-Verhältnisses, des Bündel-Wand-Abstands, der radialen Verschiebung der Stabringe untereinander sowie des Symmetriewinkels des charakteristischen Segmentes dargestellt.

### ДВУМЕРНЫЙ АНАЛИЗ ЛАМИНАРНОГО ТЕЧЕНИЯ ЖИДКОСТИ В ПУЧКАХ СТЕРЖНЕЙ ПРИ ПРОИЗВОЛЬНОМ РАСПОЛОЖЕНИИ

**Аннотация** — В данной статье описывается аналитический метод расчёта полностью развитого ламинарного продольного течения в нескольких различным образом соединённых пучках труб, свободный от ограничений, принятых в предыдущих исследованиях. Результаты представлены в виде двумерного поля скоростей и произведения  $f_t \cdot Re$  в зависимости от отношения диаметра к длине, расстояния между стержнями, радиального смещения пучков относительно друг друга и угла симметрии характеристического сегмента.